A radial-symmetric checkerboard stimulus obeying the inverse-linear cortical magnification law

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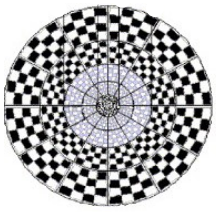
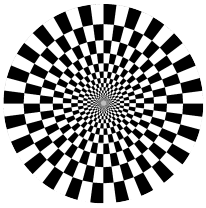
Introduction

To study the visual cortical architecture in V1 in fMRI, or diagnose visual field defects in multifocal electrophysiology, we need stimuli that compensate for the cortical magnification factor (CMF). This insures stimulating the visual field in a uniform manner. Popular stimuli were said to do that. But on closer inspection it is not clear they do, since the exact equations for the ring radii that would correspond to the inverse-linear CMF function were unknown. Here we derive these equations.



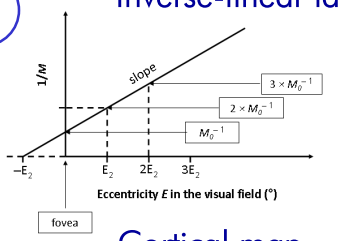
Exponentially scaled mVEP stimulus (left; Seiple et al. 2005) and fMRI stimulus (right; S. Islam 2010). Note that the center is undefined and the increase only approximate in both cases.



Terms & Definitions

1 Inverse-linear law

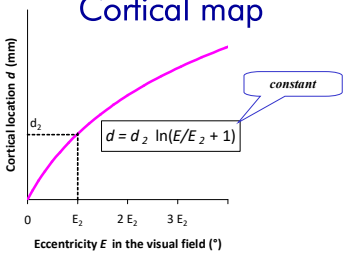
E_2
M_0
slope



CMF Function, local behaviour

2 Cortical map

d_2



Location Function, global behaviour

E_2 (in ° visual angle) – is the eccentricity where the foveal value is doubled.

d_2 (in mm cortex) – is where the retinotopic centre's value is halved

M_0 (mm/°) is the foveal cortical magnification factor (CMF).

slope refers to the 1/M vs. ecc. Function.

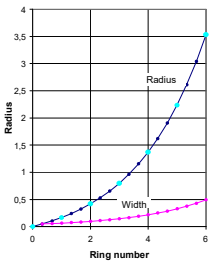
The inverse-linear *CMF* (1) translates to a logarithmic *location function* (2) in retinotopic visual areas like V1.

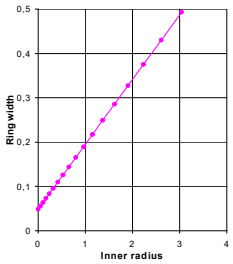
As Fischer (1973) has shown, the constant term is essential for the eq. being physically possible (Str. 2022).

Results

To counter the peripherally reduced cortical area and achieve equal cortical stimulation, a suitable stimulus needs to increase by the inverse function:

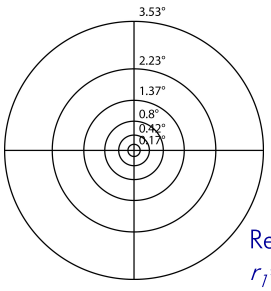
$$r_n = E_2(e^{an} - 1) \quad n \geq 0 \qquad (3)$$





Example

This exponential radius function has the unique property that the resulting ring widths w_n depend linearly on ring location.



Result for r_1=0.17°

$$a = \ln\left(\frac{r_1}{E_2} + 1\right) \qquad (4)$$

The exponential parameter *a* determines slope

A quick derivation

Let the outer radius be $r_n = A(e^{an} - 1), \; n \geq 0$ (1)

It follows $r_0 = 0$ and $r_1 = A(e^a - 1)$ (1a)

Exponent parameter a follows from (1a): $a = \ln\left(\frac{r_1}{A} + 1\right)$ (1b)

Ring width w_n, by definition: $w_n = r_n - r_{n-1} \quad n \geq 1$ (2)

Insert (1) into (2): $w_n = A(e^{an} - 1 - (e^{a(n-1)} - 1)) = A(e^{an} - e^{a(n-1)}) = Ae^{a(n-1)}(e^a - 1)$ (3)

Thus, ring width is a pure exponential function, $w_n = \alpha e^{a(n-1)}$, scaled by $\alpha = A(e^a - 1) = r_1$ (4)

$w_n = r_1 e^{a(n-1)}$ (4a)

Is ring width linearly dependent on its inner radius? I.e., what is $w_n = f(r_{n-1})$?

$r_{n-1} = A(e^{a(n-1)} - 1)$ is the inner radius (1c)

converted: $e^{a(n-1)} = \frac{r_{n-1}}{A} + 1$ (1d)

and inserted into (4a) gives $w_n = r_1\left(\frac{r_{n-1}}{A} + 1\right)$, with $r_1 = \alpha = A(e^a - 1) \quad n \geq 1$ (13)

or $w_n = A(e^a - 1)\left(\frac{r_{n-1}}{A} + 1\right)$. (13a)

That function is linear, with **intercept** $r_1 = A(e^a - 1)$ and **slope** σ: $(e^a - 1)$. Or $\sigma = \frac{r_1}{A}$. (13b)

I.e., the intercept is determined by the first radius, which, together with *A*, also determines the slope.

Comparison with CMF scaling $S = S_0\left(\frac{E}{E_2} + 1\right)$ yields $r_1 = S_0$ and $A = E_2$. (14)

Conclusion

Radii are given by (3) where the inner circle's radius r_1 is a free parameter. The exponent parameter *a* for ring width is given by cortical E_2 (eq. 4). A good value (from Schira et al. 2009) is $E_2 = 0{,}33°$.

References

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Schira MM, Tyler CW, Breakspear M, Spehar B (2009). The foveal confluence in human visual cortex. *J. Neuroscience*, 29(28), 9050–9058.
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