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Efficient estimation of sensory thresholds with ML-PEST

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Abstract—A set of C and C++ routines are described that allow the efficient estimation of sensory thresholds in psychophysical experiments using a maximum-likelihood staircase procedure. They have been used effectively in visual, auditory, gustatory, and olfactory psychophysics.

This ML-PEST package of sixteen C subroutines, two C++ classes and six demonstration programs takes its name from its use of maximum-likelihood methods, first proposed by Hall (1968), to carry out Parameter Estimation by Sequential Testing (Taylor and Creelman, 1967; Findlay, 1978). The subroutines allow an experimenter to efficiently estimate two parameters of a psychometric function in a psychophysical experiment: α, the so-called threshold parameter, and β, the steepness parameter. They also provide a statistical basis for deciding when to stop testing. More detailed descriptions of these techniques may be found in several places (Levitt, 1971; Watson and Pelli, 1983; Harvey, 1986; Treutwein, 1995).

Five psychometric functions are provided in the package: the logistic function (Eqn 1) (Berkson, 1953); the Weibull function (Eqn 2) (Weibull, 1951); the Gaussian integral function (Eqn 3) (Zelen and Severo, 1964); the cumulative Poisson function (Eqn 4) (Hecht ***et al.,*** 1942); and the step function (Eqn 5) (Simpson, 1989).

$$P(x) = \gamma + \left((1-\gamma)\cdot\left(\frac{1}{1+\frac{1}{(SL)^{\beta}}}\right)\right), \tag{1}$$

$$P(x) = \gamma + \left((1-\gamma)\,.\,\left(1-\exp\left(-(SL)^{\beta}\right)\right)\right), \tag{2}$$

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$$P(x) = \gamma + \left((1-\gamma)\cdot\int_{-\infty}^{x}\left(\frac{\beta}{\sqrt{2\pi}}\right)\cdot\left(\exp\left(-\frac{\left(\log_{10}\left((SL)^{\beta}\right)\right)^{2}}{2}\right)\right)\mathrm{d}x\right), \quad (3)$$

$$P(x) = \gamma + 1 \quad - \gamma) \left(1 - \sum_{k=0}^{\beta-1}\frac{(\exp(-\mathrm{SL})\,.\,(SL)^{k})}{k!}\right)\Bigg), \quad (4)$$

$$P(x) = \gamma + \left((1-\gamma)\cdot\begin{pmatrix}0 & \text{if } x < \alpha\\ 1 & \text{if } x \geqslant \alpha\end{pmatrix}\right). \quad (5)$$

These functions are completely specified by three parameters: a , the stimulus intensity at which the slope of the function is maximum (the 'threshold' parameter); β, the steepness of the function; and γ, the probability of a correct response due to chance alone. The step function has infinite β. The term SL (stimulus level) in the equations is computed from a and the stimulus value x: $\mathrm{SL} = (x/\alpha)$.

Note that, with the exception of the step, these functions are consistent with the key assumption of signal detection theory that there is no sensory threshold, i.e. no stimulus level below which the response probability is at chance level (Swets, 1961; Swets ***et*** *al.,* 1961; Green and Swets, 1966, 1974; Krantz, 1969). In fact the logistic, Gaussian integral, and cumulative Poisson functions are undefined for a stimulus value of zero. Data from forced-choice experiments are usually well-described by a Weibull function (Nachmias, 1981; Harvey, 1986) although the logistic and Gaussian integral functions are often equally good. When visual detection is based on a few quanta (Hecht *et al.,* 1942; Sakitt, 1972; Nerger and Cicerone, 1992) the cumulative Poisson function is appropriate. The step function is of little use in psychophysics (Watson and Fitzhugh, 1990).

Since the original ML-PEST routines were described (Harvey, 1986), laboratory computers have become much faster, the C++ computer programming language has been developed (Stroustrup, 1991), and bootstrap statistical techniques for estimating parameters and their confidence intervals from scant data have become available (Efron and Tibshirani, 1985; Foster and Bischof, 1991, 1997). I have therefore written two C++ classes that encapsulate and extend the functionality of the original routines.

C++ Class TMLpest. This class keeps a running estimate of the value of alpha and its confidence interval during psychophysical testing. The public member functions of this class are given in Appendix I. The class constructor specifies the range of candidate values that are to be tested for being alpha. For sinusoidal grating stimuli, I usually use log contrasts ranging from −3.00 to 0.00 in 0.01 log-unit steps. Three separate likelihood arrays are maintained. The posterior likelihood includes the *a priori* estimate of what the value of alpha might be before any data are collected in the experiment. The default *a priori* likelihood function is Gaussian. The candidate alpha having the highest posterior likelihood is used to pick the stimulus for the next trial (see below). It is important to use the posterior estimate of alpha to control the stimulus presentation because the likelihood estimate of alpha is very unstable during the first 10 or so trials. The likelihood array is computed from the results of the current psychophysical trials. The candidate alpha having the highest likelihood is the best estimate of alpha. The third array is used to detect malingering. These likelihoods are based on the hypothesis that the subject is choosing the wrong answer on each trial, not the correct answer. If the maximum-likelihood of this array is higher than the maximum value of the normal likelihood array, the observer may be malingering.

C++ *Class* TMLstim. This class holds the stimulus values that are being used in a psychophysical experiment. The public member functions of this class are given in Appendix **II**. For example, the stimuli could be a series of contrasts or spatial frequency increments (**A**f/f). The experimenter determines what values are stored in the class. The class constructor specifies how many test stimuli are to be stored and then the specific values can be stored using the *SetStimulus*() member function.

For maximum efficiency in estimating alpha, it is desirable to present a stimulus on the next trial that is equal to the best estimate of alpha (Taylor, 1971). Often, however, it is not possible to generate an actual stimulus having the required value. In taste and smell testing, for example, there may be only 20 concentrations in *0.25* log unit steps. When the estimated alpha lies between two realizeable stimulus values, *TMLstim* has a member function to choose either the higher or lower of the two stimuli with a probability that is proportional to the closeness of the alpha value. Two other member functions estimate alpha and beta and their confidence intervals using a maximum-likelihood method based on the method of Powell (Press *et al., 1992)* and based on a bootstrap fitting procedure (Foster and Bischof, 1991). One or both of these methods may be used after the psychophysical trials have ended to obtain better estimates of alpha and beta and their confidence intervals than is possible during the testing.

These two classes simplify the writing of programs to run psychophysical experiments because various housekeeping chores are handled automatically. It is easy to have multiple staircases in the same experiment by creating as many new instances of the *TMLpest* and *TMLstim* objects as you need. Pointers to these objects can be put into an array and the array can be randomly sampled to create multiple random staircases.

Four demonstration programs written in ***C*** are provided to illustrate the use of the C subroutines. Programs *sim1.c* and *sim2.c* simulate a psychophysical observer in a constrast sensitivity experiment using a 2AFC detection paradigm. Program *sim3.c* simulates an experiment in which a complete psychometric function is measured using the method of constant stimuli in a 2AFC paradigm. Following collection of the data, the maximum-likelihood curve fitting is used to compute the best-fitting values of alpha and beta. Program *mlpfit.c* computes a maximum-likelihood fit of user-provided data to each of the five psychometric functions.

Three programs written in C++ illustrate the use of the C++ classes: program *sim1.cp* has the same functionality as *sim1.c* but uses the two C++ classes. Likewise, *sim3.cp* has the same functionality as *sim3.c* but uses the the *TMLstim* C++ class. Program *mlpfit.cp* uses the *TMLstim* class to compute both the maximum-likelihood and the bootstrap fits of psychometric functions to data. All source code is written in

ANSI **C** and **C++** and has been extensively tested on various Macintosh systems using both Symantec and Metrowerks C/C++ compilers. They have also been compiled and run using Borland C/C++ for MS-DOS systems, VAX and unix systems with no modifications.

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Availability

The ML-PEST package of **C** and **C++** routines and programs is available free of charge from the author's public WWW page.

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